BANKED TURNS AND CURVES

Unit 5: Circular Motion

REVIEW

On a horizontal section of track, an 81-kg skier makes a turn at a velocity of 50 km/h. What is the magnitude of her centripetal force if the radius of curvature of her trajectory is 22 m?

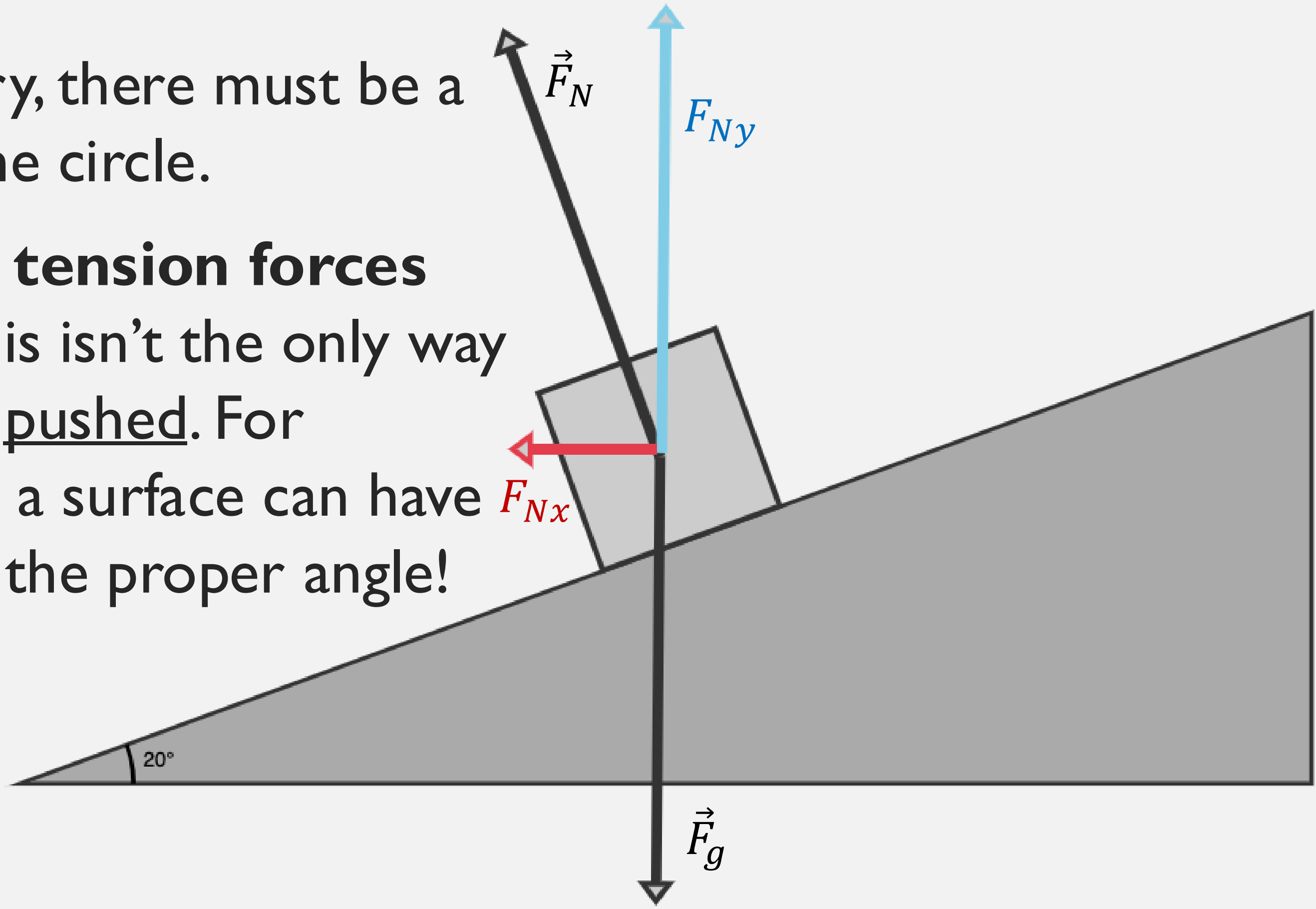
$$m = 81\ \text{kg}$$
$$v = 50\ \text{km/h} = 13.9\ \text{m/s}$$
$$r = 22\ \text{m}$$

$$F_c = \frac{mv^2}{r} = \frac{(81\ \text{kg})(13.9\ \text{m/s})^2}{22\ \text{m}}$$
$$= 711\ \text{N}$$

COMBINING CENTRIPETAL FORCE WITH INCLINED PLANES

For an object to travel in a circular trajectory, there must be a **centripetal force** towards the centre of the circle.

So far, we've seen circular motion caused by **tension forces** pulling the object towards the centre, but this isn't the only way that a force can affect an object! It could be pushed. For example, the **normal force** of contact with a surface can have a vector component in the x-direction with the proper angle!



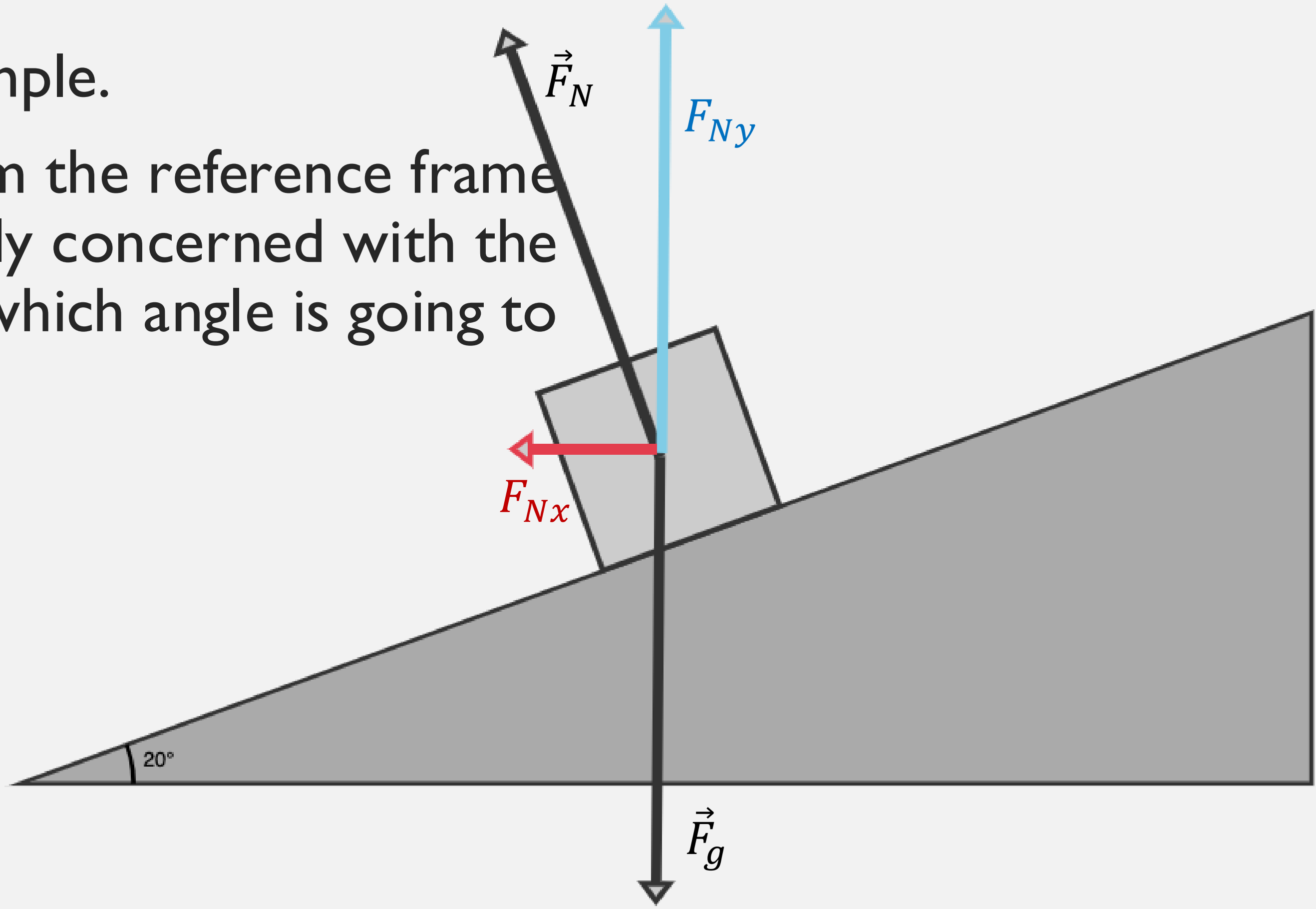
COMBINING CENTRIPETAL FORCE WITH INCLINED PLANES

Take the inclined plane to the right, for example.

Normally, we would look at the vectors from the reference frame of the ramp's angle, but here we are primarily concerned with the x-direction, because we want to figure out which angle is going to make this object turn in a **circle**!

$$F_{Nx} = F_c$$

$$\Rightarrow F_N \sin\theta = \frac{mv^2}{r}$$



COMBINING CENTRIPETAL FORCE WITH INCLINED PLANES

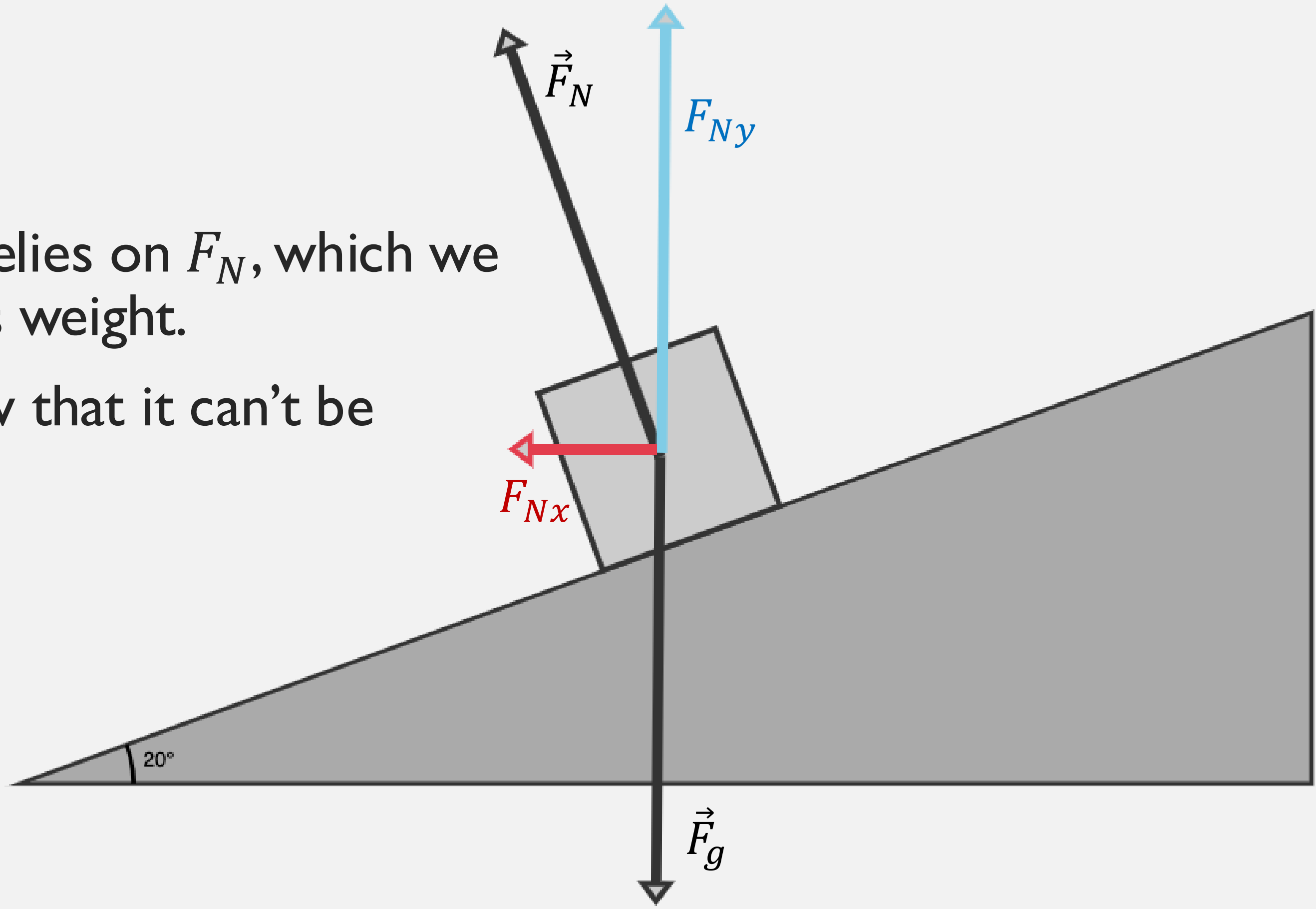
$$\sin\theta = \frac{mv^2}{rF_N}$$

Here we have an equation for the angle, but it relies on F_N, which we don't know! But we can find it using the object's weight.

Since the object is on a **circular path**, we know that it can't be gaining or losing any height on the track.

$$\therefore F_{Ny} = mg$$

$$\Rightarrow F_N \cos\theta = mg$$



COMBINING CENTRIPETAL FORCE WITH INCLINED PLANES

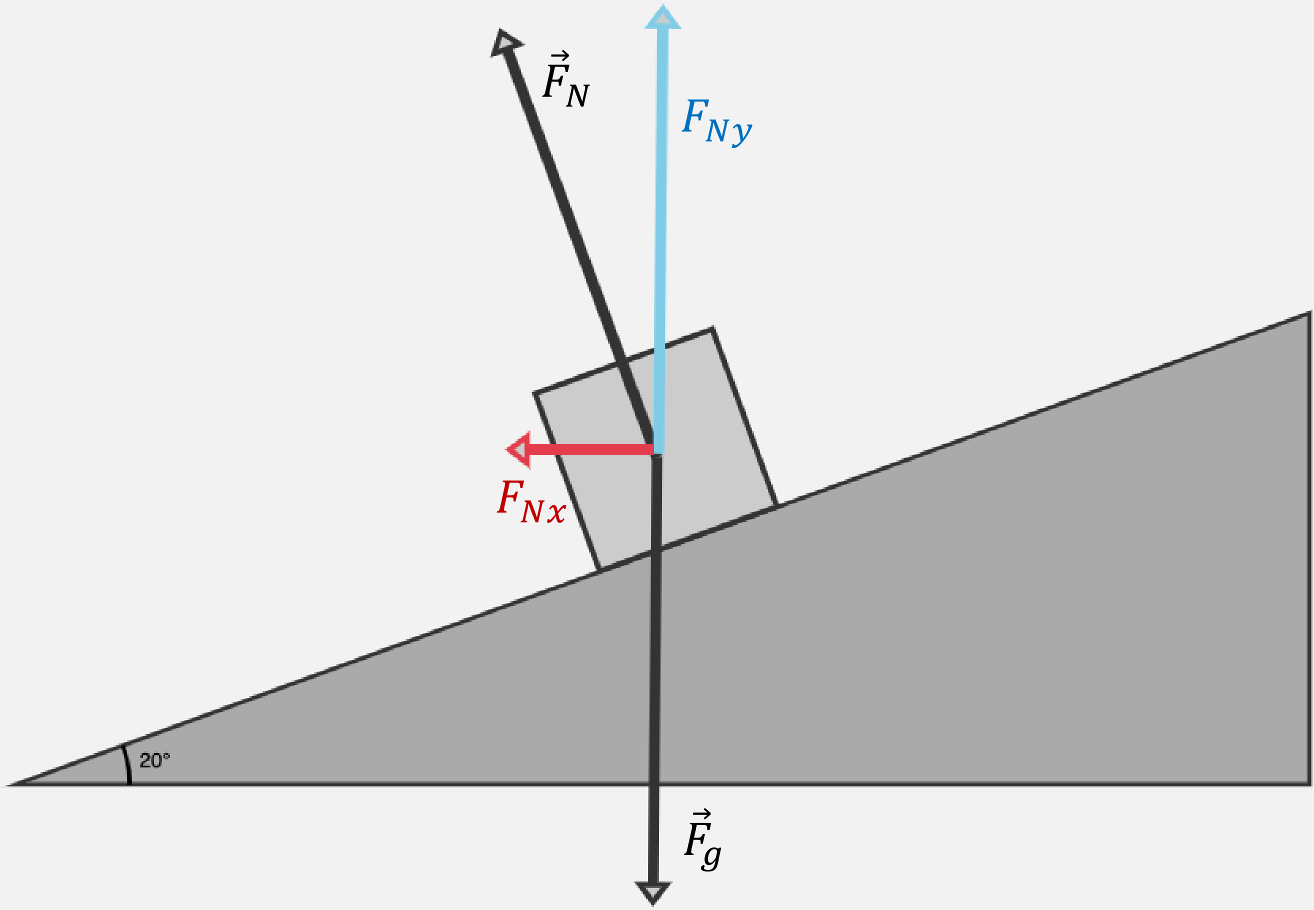
$$\sin\theta = \frac{mv^2}{rF_N}$$

$$F_N = \frac{mg}{\cos\theta}$$

Combine the two equations:

$$\sin\theta = \frac{mv^2}{r\frac{mg}{\cos\theta}}$$

$$\Rightarrow \tan\theta = \frac{v^2}{rg}$$



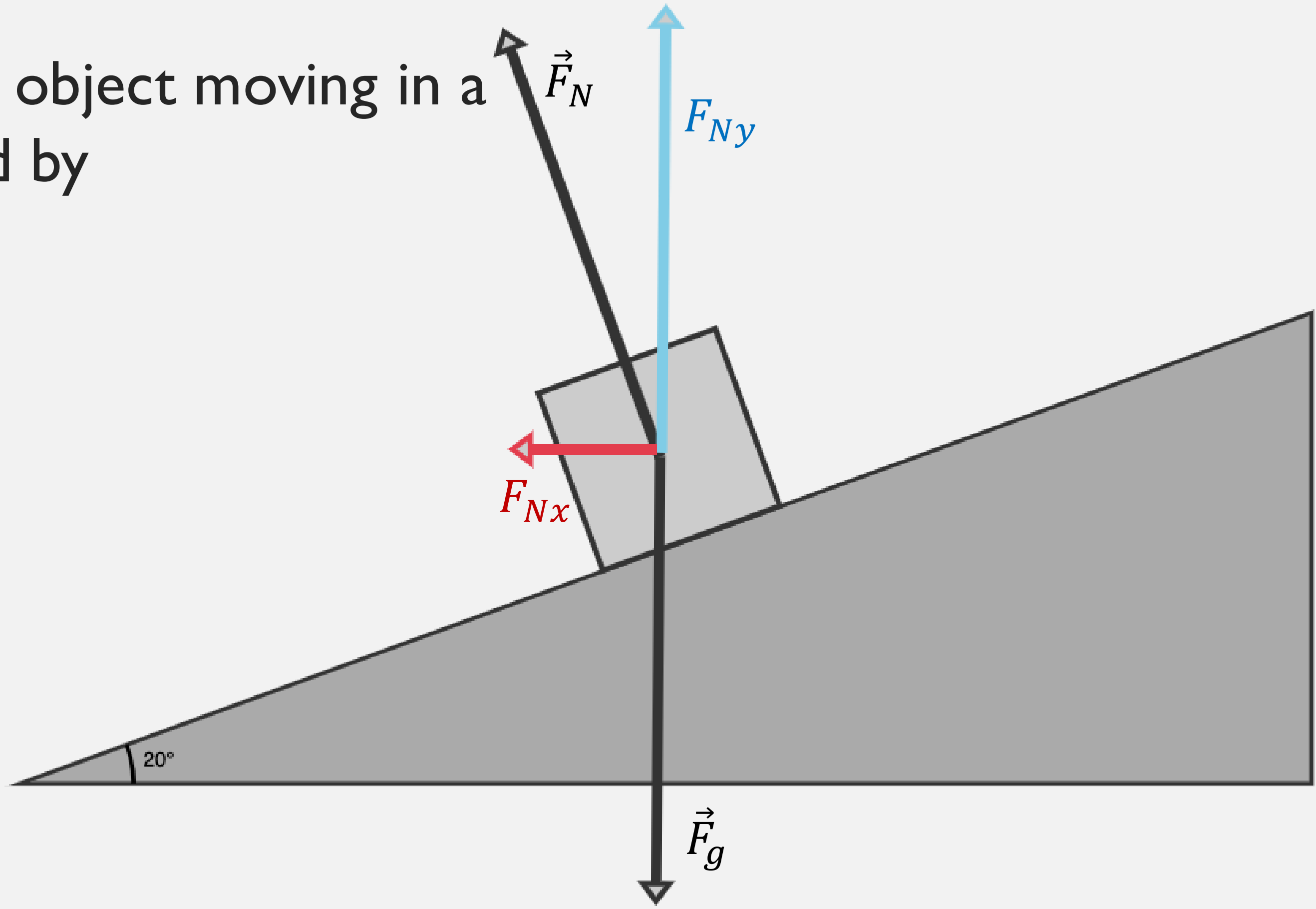
COMBINING CENTRIPETAL FORCE WITH INCLINED PLANES

The angle of a banked turn that will keep an object moving in a circular path at a constant velocity is defined by

$$\tan\theta = \frac{v^2}{rg}$$

- v is the object's velocity along the turn
- r is the turning radius

One note: this is still affected by the maximum angle defined by $\mu_s = \tan\theta$. That means that if $\frac{v^2}{rg} \geq \mu_s$, the object will still slide down the ramp!



BANKED TURNS

BANKED TURNS

Based on your understanding of forces, what is the benefit of banking?

How does this help drivers to turn better than a flat track?

TURNING

On a flat road, turning is accomplished via **friction**.

When a car's wheels **turn**, they move the direction of force towards a new angle. This also means that in the forward direction, the wheels are being affected much more by the friction between the rubber and the road.

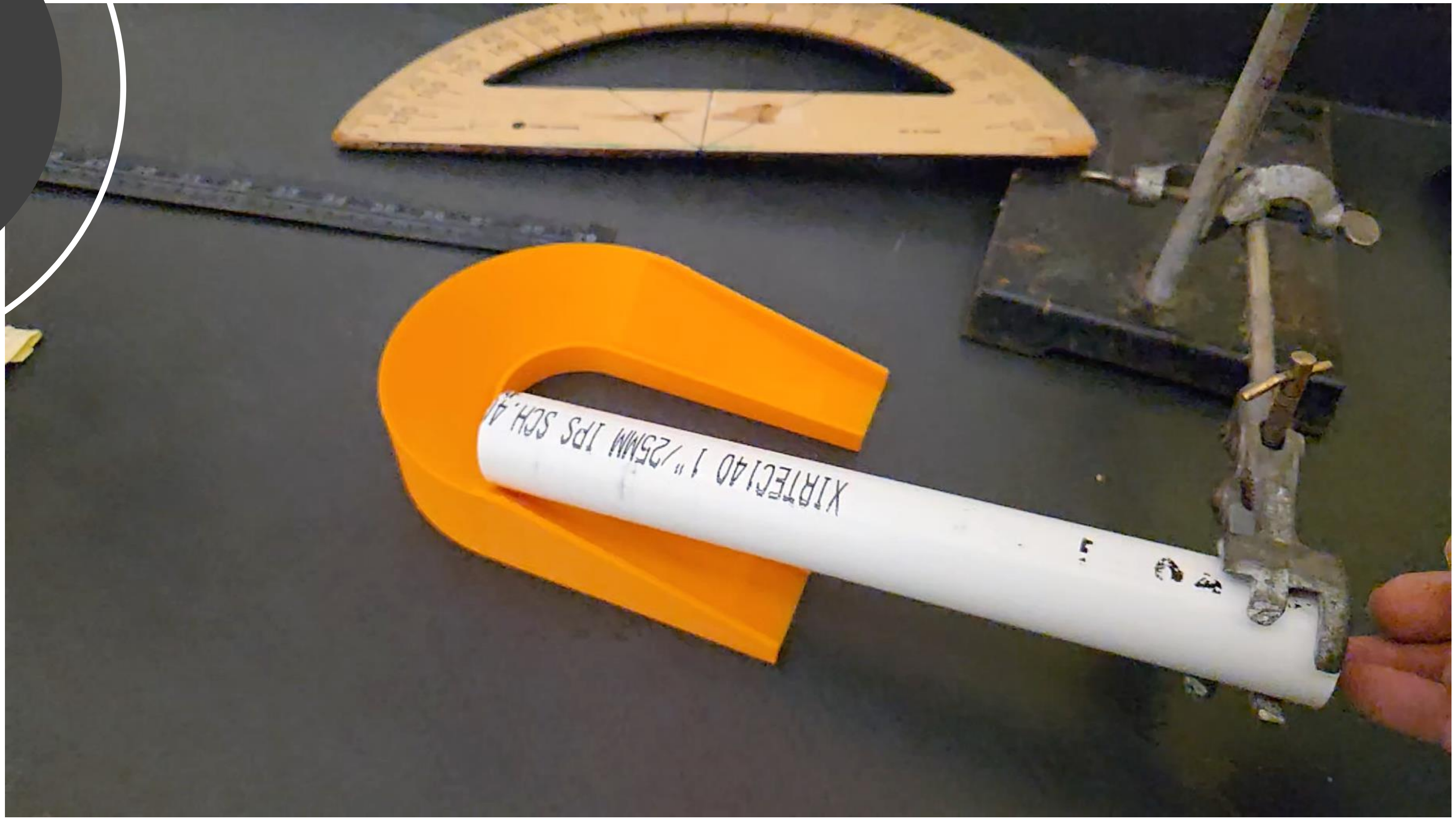
TURNING

This means that, at high speeds, two things will occur due to the friction on the tires:

- Major reduction in speed (*not ideal for a race!*)
- Wearing of tires (*wasteful, and means more frequent pit stops*)

To conserve tire rubber and speed, banked turns effectively make it so that drivers can turn their wheels less while still turning their car!

ASSIGNMENT: DESIGNING BANKED TURNS



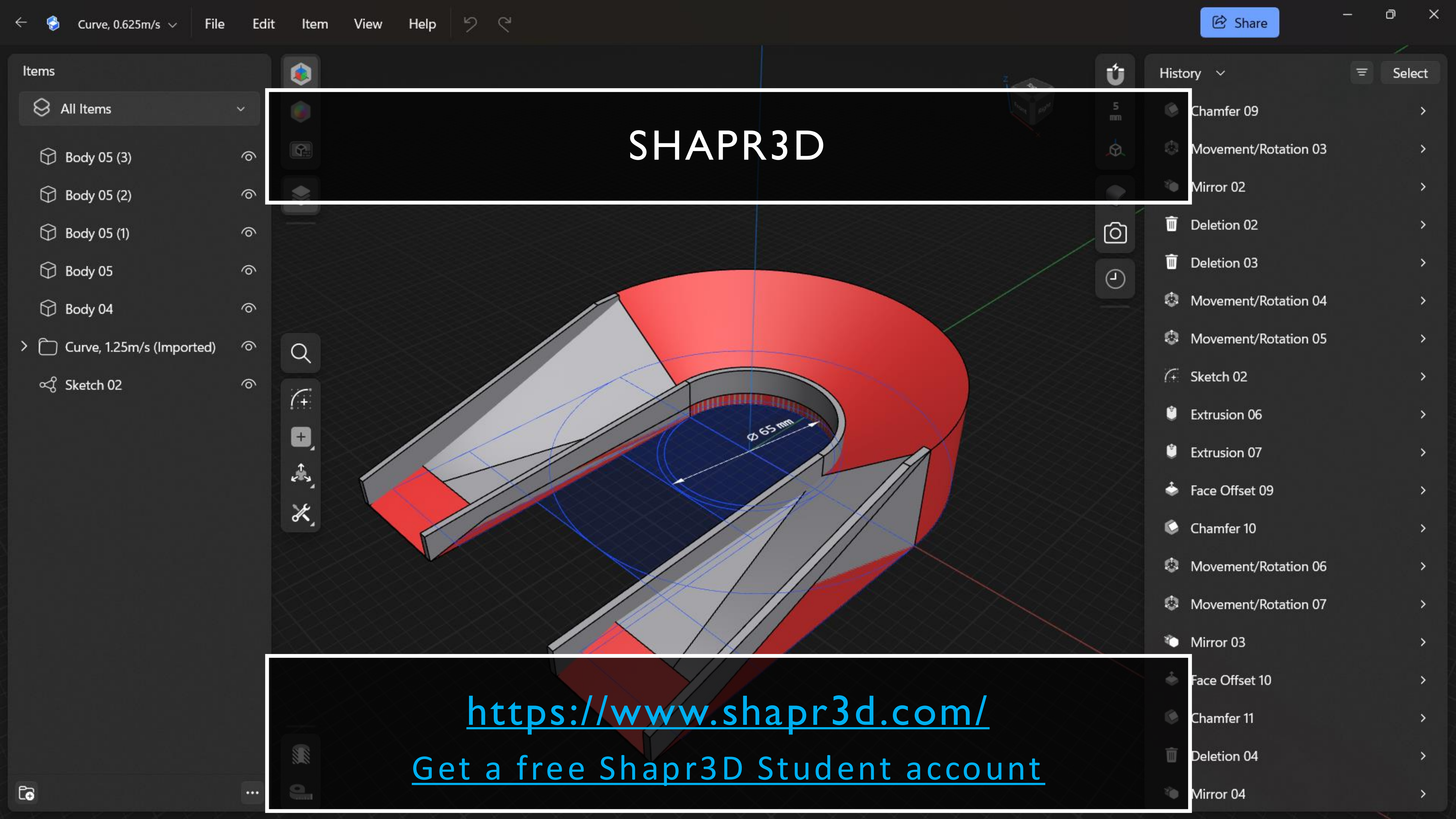
ASSIGNMENT: DESIGNING BANKED TURNS

Your task is to design and 3D print a 180° turn that will allow a marble of a given velocity to travel down the **middle** of the track and **turn all the way around**.

- I recommend a **turning radius** of 5 cm, but it can be whatever you'd like, so long as it fits on the printing bed (25.6 cm long × 25.6 cm wide × 25.6 cm high)
- Each group will be given a different **speed**.
- Combine your radius and speed to find the angle.
- Think sustainably! When printing your designs, think about how the forces of the marble will affect the printed object. How can you reduce the material you use while still maintaining structural integrity?
- The closer your marble stays to the centre, the better!

MARKING CATEGORIES

Design	Was your design built to the proper angle for your track? Are the radius and height reasonable? Do you have an easy entry point for the marble? **To Submit:** Your Shapr3D file or a technical drawing, your calculations
Efficiency	How much filament did you use? What infill level and what type of internal supports did you choose? Can your print support the force of the marble? **To Submit:** A brief justification for your slicing choices (one or two paragraphs, discussing sustainability and integrity)
Testing	Does it work? Does the marble fly off the edge or roll down the ramp, or does it stick to the centre and travel all the way around? **To Submit:** A brief discussion of your observations; explain what happened during your test. What could be improved?


SHAPR3D
https://www.shapr3d.com/
Get a free Shapr3D Student account
Curve, 0.625m/s
File
Edit
Item
View
Help
Share
Items
All Items
Body 05 (3)
Body 05 (2)
Body 05 (1)
Body 05
Body 04
Curve, 1.25m/s (Imported)
Sketch 02
Ø 65 mm
History
Select
Chamfer 09
Movement/Rotation 03
Mirror 02
Deletion 02
Deletion 03
Movement/Rotation 04
Movement/Rotation 05
Sketch 02
Extrusion 06
Extrusion 07
Face Offset 09
Chamfer 10
Movement/Rotation 06
Movement/Rotation 07
Mirror 03
Face Offset 10
Chamfer 11
Deletion 04
Mirror 04

3D DESIGN TIPS

Do your calculations before you start designing. Find your angle, then use that to find your ideal dimensions. In the file I've provided, the track is 4 cm wide. How tall will the curve have to be to get the right angle?

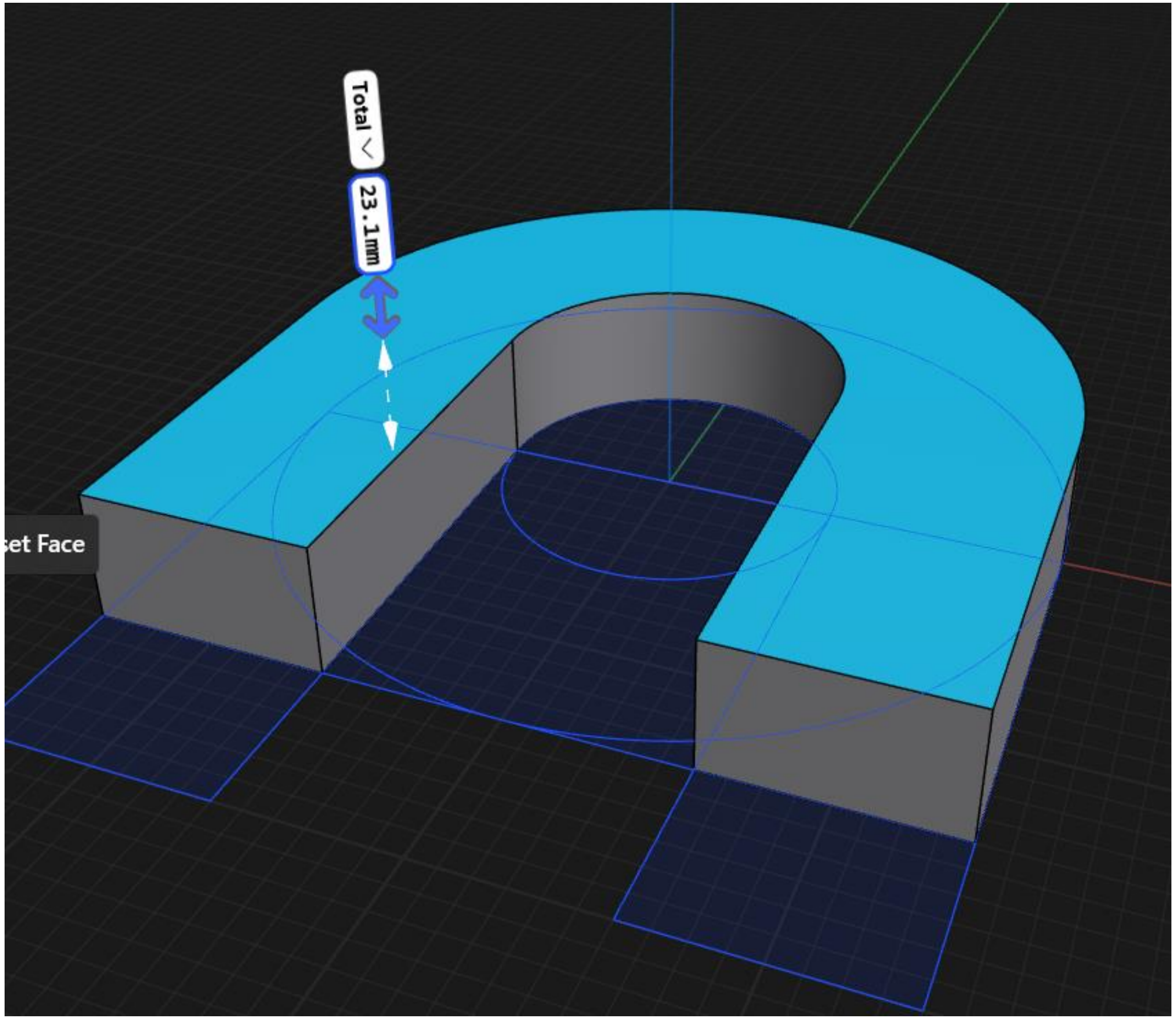
If this is your first time, **do the tutorial**! It's helpful for the basic functions.

Start with the base project file posted on Classroom. Import it into your file.

The best way to get the precise angle you need is the Chamfer tool (a chamfer is an angled cut along an edge. By default the Auto-chamfer will cut at an angle of 45°, but using 2-Distance Chamfer, you can get whatever angle you need!

CHAMFER

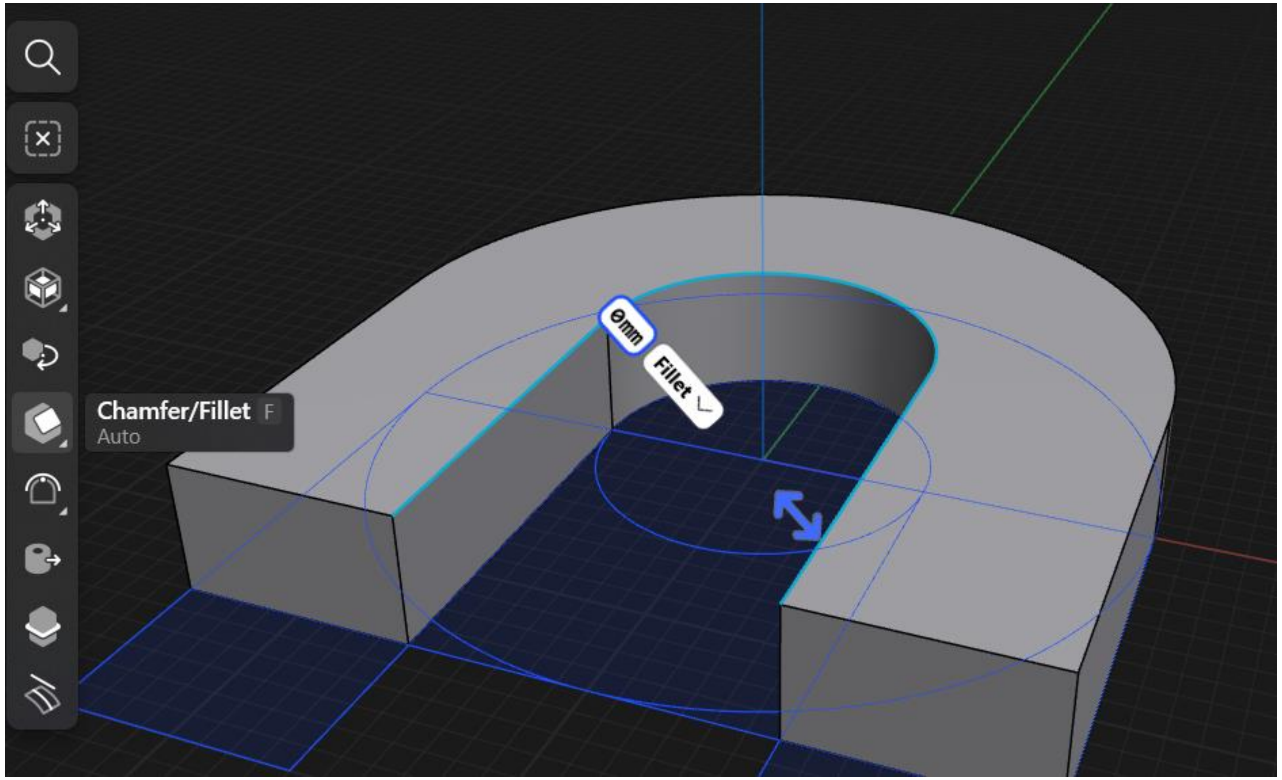
If I want a track angle of 30° and my track is 4 cm across, I need a height of 2.31, or 23.1 mm.



CHAMFER

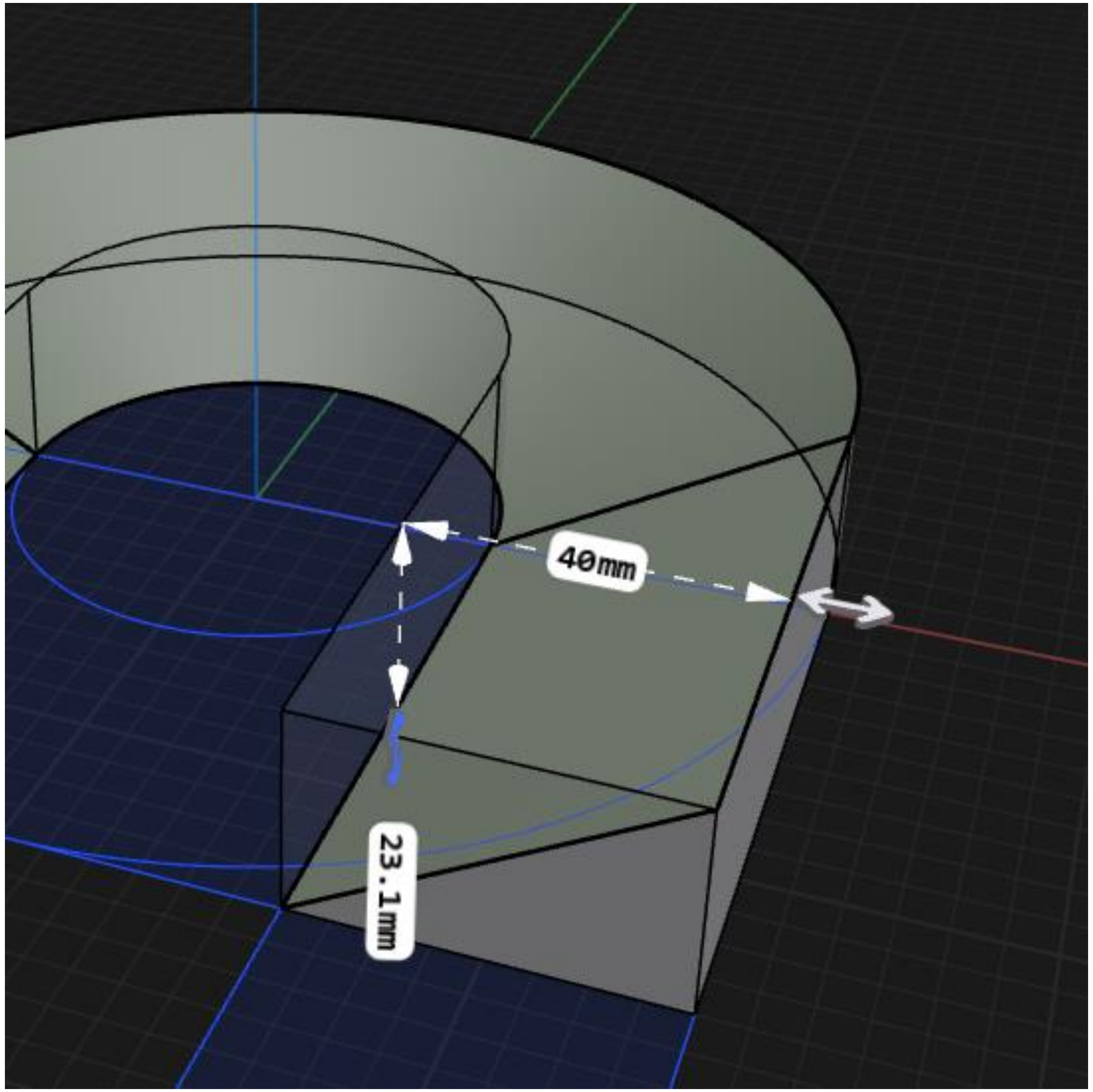
Selecting the edges of the solid automatically brings up the Chamfer/Fillet tool.

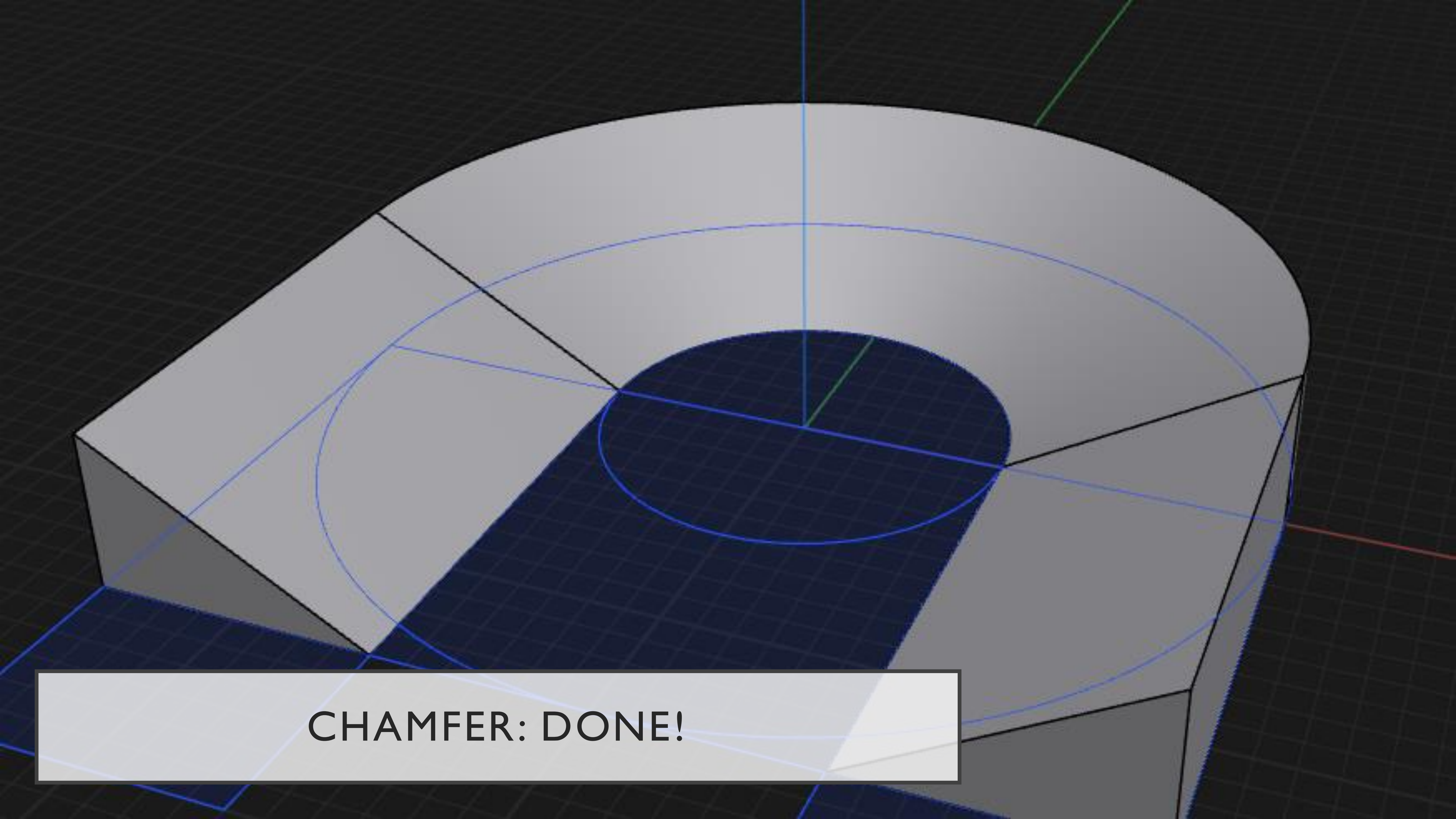
Click on Chamfer/Fillet to change to 2-Distance Chamfer.



CHAMFER

Manually input the distances in the x and y directions, and you'll have the angle at that edge.





CHAMFER: DONE!